※請將答案填入試卷內所附的表格中。

1. (10 points) Let there be z functions from $\{\text{"A"}, \text{"B"}\}^3$ to $\{1,2,3\}$. Calculate z mod 7: _____.

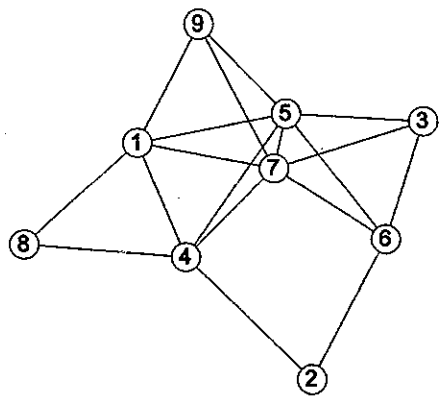
2. (10 points) There are _____ satisfying assignments to

$$(\neg w \wedge x \wedge \neg y \wedge \neg z) \vee (\neg w \wedge \neg x \wedge y) \vee (\neg w \wedge \neg x \wedge z).$$

3. (10 points) Calculate Euler's phi function (also called totient function) $\Phi(90)$: _____.

4. (10 points) Derive the solution for a_n that satisfies the recurrence equation $a_n = -a_{n-1} + 12a_{n-2}$ with $a_0 = 3$ and $a_1 = 2$: _____.

5. (10 points) For the following graph: (4 points) Its chromatic number is _____ (the smallest number of colors needed to color the nodes). (4 points) The size of the maximum independent set is _____. (1 point) Is it Eulerian _____ (yes/no)? (1 point) Is it Hamiltonian _____ (yes/no)?



見背面

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6. (10 points). Consider a system of linear equations $Ax = b$ with the following augmented coefficient matrix:

$$[A \mid b] = \left(\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 0 \\ 1 & -1 & -3 & 2 \end{array}\right).$$

Let

$$S = \operatorname*{argmin}_{x \in \mathbb{C}^3} \|Ax - b\|.$$

That is, S consists of the vectors $x \in \mathbb{C}^3$ that minimize the standard norm (also known as 2-norm or ℓ^2-norm) of the vector $Ax - b \in \mathbb{C}^3$. The square of the minimum standard norm of the vectors in S is _____.

7. (10 points) Consider the following sets R and S of quadruples of rational numbers:

$$R = \left\{\left(\frac{1}{3}, \frac{-2}{3}, \frac{1}{3}, 0\right), \left(\frac{-3}{2}, 0, \frac{3}{2}, 0\right)\right\}$$
$$S = \{(1,0,-1,0), (1,-1,0,0), (-1,1,0,1), (1,0,-1,1)\}.$$

If V (respectively, W) is the subspace of the rational quadruples spanned by R (respectively, S), then the subspace U of the rational quadruples satisfying the following two conditions

$W = U \oplus V$	(i.e., W is the direct sum of U and V)
$U \perp V$	(i.e., U is orthogonal to V with respect to the standard inner-product of $\mathbb{Q}$)

is _____.

8. (10 points) Let F be the field $(\{0,1,2\}, +, \cdot)$, where $+$ is the mod-3 addition and $\cdot$ is the mod-3 multiplication. If the polynomial

$$p(x) = a + bx + cx^2 \in \mathbb{P}(F)$$

satisfies $p(i) = i^3 + 2$ for each $i \in F$, then (a, b, c) is _____.

9. (10 points) If the characteristic polynomial of a linear operator T on a complex inner-product space V is

$$f_T(x) = x^4 - 2x^2 + 1,$$

then the rank(S) of the linear operator

$$S = T^6 - 2T^5 - T^4 + 4T^3 - T^2 - 2T$$

in terms of rank(T) is _____.

10. (10 points) Let U and V be vector spaces over the scalar field $\mathbb{R}$ of real numbers, where

- U consists of the 2×2 self-adjoint complex matrices and
- V consists of the 3×3 real matrices.

If

$$S = \{2, 0, 2, 4, 0, 2, 0, 3\}$$

is a multi-set consisting of the nonnegative real numbers

$$|\sigma - 1|$$

for all nonzero singular values σ of T, then the product of all singular values of the adjoint of the pseudo-inverse of T, is _____.

試題隨卷繳回