國立成功大學

111學年度碩士班招生考試試題

編　號：179

系　所：電機工程學系

科　目：離散數學

日　期：0219

節　次：第 3 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%) Assume A and B are independent events with $P(A) = 0.2$ and $P(B) = 0.3$. Let C denote the event that **none** of the events A and B occurs, and let D be the event that **exactly one** of the events A and B occurs.
   (a) Find $P(C)$.
   (b) Find $P(D)$.
   (c) Find $P(A \mid D)$.
   (d) Find $P(A \mid D^c)$.

2. (15%) Consider the set $S = \mathbf{Z}$, where $x \sim y$ if and only if $2 \mid (x + y)$.
   (a) List six numbers that are related to x=2
   (b) Prove that $\sim$ is an equivalence relation on S.
   (c) Show all equivalence classes.

3. (10%) Suppose each of the three vertices of a triangle is colored at random either black or white, with equal probability for each color. What is the expected number of edges of the triangle that have both endpoints of the same color?

4. (10%) Give an example of a relation R such that its transitive closure R^* satisfies $R^* = R \cup R^2 \cup R^3$, but $R^* \neq R \cup R^2$.

5. (10%) Recall that the Fibonacci numbers are defined by $f_0 = 0$, $f_1 = 1$, and $f_n = f_{n-2} + f_{n-1}$ for $n \geq 2$. Prove that
   $f_0 f_1 + f_1 f_2 + \cdots + f_{2n-1} f_{2n} = f_{2n}^2$ for any positive integer n.

6. (10%) Suppose we randomly choose nonnegative integers x_1, x_2, x_3, and x_4 that solve the equation $x_1 + x_2 + x_3 + x_4 = 8$. Here we assume that each solution has an equal probability of being chosen. Given that at least one of x_1 and x_2 is equal to 1, what is the probability that $x_1 = 1$.

7. (15%) Give a recursive definition of the set of bit strings that have the same number of zeros and ones.

8. (10%) Use generating functions to determine the number of different ways 10 identical balloons can be given to four children if each child receives at least two balloons. *(Note: The children are distinguishable)*