國立成功大學

110學年度碩士班招生考試試題

編　號：180

系　所：電機工程學系

科　目：離散數學

日　期：0202

節　次：第 3 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%) Define a relation R on **S** x **S** as follows:

   For all ordered pair (a,b) and (c,d) in **S** x **S**

   (a,b) R (c,d) ⇔ either $a < c$ or both $a=c$ and $b \le d$

   (a) (10%) Let S be the set of all real numbers. Show that R is a partial order relation on S.

   (b) (10%) Let S = {0,1} , draw the Hasse Diagram for R. And Name all minimal elements, if any exist.

2. (10%) What is the number of 'NCKU's printed by the pseudocode below?

```
for i from 1 to 10
  for j from i + 1 to 10
    for k from i + 2 to j - 2
      print "NCKU"
```

3. (10%) A circular necklace consists of 5 beads, each of the beads is either red, blue, or green. (a)What is the number of different necklaces if rotations of a necklace are considered the same? For example, the necklace RBBGR is the same as the necklaces BBGRR and BGRRB, but not the same as RGBBR. (b) How about with the following change: There are 7 beads instead of 5, and 4 colors instead of 3.

4. (10%) A full binary tree is a rooted binary tree where every vertex has either two children or no children. Let B_n be the number of full binary trees with n leaves, e.g. $B_4 = 5$ as:



   Find a recurrence relation of B_n. Explain your answer.

5. (10%) Show that every positive integer has a multiple which consists of 0 and 7 only. For example, 70 is a multiple of 2, 777 is a multiple of 3.

6. (10%) Use the definitions of big-Omega, big-*O*, and big-Theta to show that $5487x^4 + 87x^3 + 0487$ is $\Theta(x^4)$.

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7. (10%) Suppose you run a day care for an office building and there are seven children A, B, C, D, E, F, G. You need to assign a locker where each child's parent can put the child's food. The children come and leave so they are not all there at the same time. You have 1 hour time slots starting 7:00 am to 12:00 pm. A star in the following table means a child is present at that time. (a)What is the minimum number of lockers you need to prepare? (b)What's your plan to assign the lockers? Show your steps.

	A	B	C	D	E	F	G
7:00	*			*	*		
8:00	*	*	*				
9:00	*		*	*		*	
10:00	*		*			*	*
11:00	*					*	*
12:00	*				*		

8. (20%) Suppose a certain animal has DNA composed of five symbols, denoted by {a,b,c,d,e}, instead of the four components in earthling DNA. The four pairs cd, ce, ed, and ee never occur consecutively in a string of their DNA, but any string without forbidden pairs is valid. (Thus, bbcda is forbidden but bbdca is valid.) We're curious how many DNA strings of length n are possible? Let a_n stand for the number of valid DNA strings that end in either c or e, and let b_n stand for the number of valid DNA strings that end in either a, b, or d.

   (a) (5%) Give a recurrence equation for a_n in terms of $a_{n-1}, a_{n-2}, \cdots, a_1$ and $b_{n-1}, b_{n-2}, \cdots, b_1$. Explain your answer.

   (b) (5%) Give a recurrence equation for b_n in terms of $a_{n-1}, a_{n-2}, \cdots, a_1$ and $b_{n-1}, b_{n-2}, \cdots, b_1$. Explain your answer.

   (c) (5%) Hence, or otherwise, give a recurrence equation for b_n in terms of $b_{n-1}, b_{n-2}, \cdots, b_1$. Explain your answer.

   (d) (5%) Find the closed form for b_n. Show your steps.